Multi-Attribute Indexing

Database Systems Concepts
Silberschatz/ Korth
Sec. 11.9

Fundamentals of Database Systems
Elmasri/Navathe
Sec. 6.4

The Design and Analysis of Spatial Data Structures
Samet

Overview

• Query Types
  − Exact Match
  − Partial Match
  − Range Match
• Strategies for Querying and Indexing
• Composite Key Index
• Visualising the Index Structure
• Multi-Attribute Index
• Bitmap Index
There are three types of query:
- Exact Match Query
- Partial Match Query
- Range Match Query

Query type determines:
- Search strategy
- Indexes used

Exact Match Queries
• The query matches with a single tuple.
  e.g.
  account(account_number, customer_name, branch_name, balance)

  SELECT *
  FROM account
  WHERE account_number = 123456789;

  Only one tuple has account_number=123456789

• An exact match query specifies enough information about the content of the tuple to allow the DBMS to identify a single tuple.
• This is normally achieved by specifying a primary key value.
• If there is an index on the primary key then it may be used to answer the query.

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• This is normally achieved by specifying a primary key value.
• If there is an index on the primary key then it may be used to answer the query.
Partial Match Queries

- The query matches with a subset of tuples.
  e.g.
  
  `SELECT * 
  FROM account 
  WHERE branch_name = 'Bournemouth';`
  
  Many tuples may have `branch_name='Bournemouth'`

- A partial match query specifies enough information about the content of the tuple to allow the DBMS to identify a set of tuples.
- A partial match query occurs when a non-key attribute is included in the WHERE clause.
- If there is an index on the attribute then it may be used to answer the query. However, the index will contain duplicate values (this is possible in a B+-Tree).

Range Match Queries

- The query matches with a range of tuples.
  e.g.
  
  `SELECT * 
  FROM account 
  WHERE balance > 1500;`
  
  Many tuples may have `balance > 1500`

- A range match query specifies the lower/upper limits of a range of values in a set of tuples.
- A range match query can occur with a key or non-key attribute in the WHERE clause.
- If there is an index on the attribute then it may be used to answer the query. A B+-Tree is suitable for certain types of range query.
Overview

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  – Exact Match
  – Partial Match
  – Range Match

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  • Composite Key Index
  • Visualising the Index Structure
  • Multi-Attribute Index
  • Bitmap Index

Strategies for Indexing 1

• Given a query:
  SELECT *
  FROM account
  WHERE branch_name = 'Bournemouth'
  AND balance = 1000;

• Strategy:
  1. Use index to search for branch_name='Bournemouth'
  2. Search result of step (1) for balance=1000

• The query is a partial match query looking for all accounts in the Bournemouth branch with a balance of 1000.
  • The first strategy for answering this query is:
    1. Use an index on branch_name to retrieve the set of tuples with branch_name='Bournemouth'.
    2. Search the set of tuples, resulting from step 1, for those tuples with balance=1000.

• Problem
  • If many records contain branch_name='Bournemouth', step 2 will involve searching many records (a serial/sequential search).

Ref: Silberschatz, sec 11.9
Strategies for Indexing 2

- Given a query:
  
  ```sql
  SELECT * 
  FROM account 
  WHERE branch_name = 'Bournemouth' 
  AND balance = 1000;
  ```

- Strategy:
  1. Use index to search for balance=1000
  2. Search result of step (1) for branch_name='Bournemouth'

- The query is a partial match query looking for all accounts in the Bournemouth branch with a balance of 1000.
- The second strategy for answering this query is:
  1. Use an index on balance to retrieve the set of tuples with balance=1000.
  2. Search the set of tuples, resulting from step 1, for those tuples with branch_name='Bournemouth'.
- **Problem**
  - If many records contain balance=1000, step 2 will involve searching many records (a serial/sequential search).

Ref: Silberschatz, sec 11.9

Strategies for Indexing 3

- Given a query:
  
  ```sql
  SELECT * 
  FROM account 
  WHERE branch_name = 'Bournemouth' 
  AND balance = 1000;
  ```

- Strategy:
  1. Use index to search for branch_name = 'Bournemouth'
  2. Use index to search for balance=1000
  3. Take the intersection of the results of steps (1) and (2)

- The query is a partial match query looking for all accounts in the Bournemouth branch with a balance of 1000.
- The third strategy for answering this query is:
  1. Use an index on branch_name to retrieve the set of tuples with branch_name='Bournemouth'.
  2. Use an index on balance to retrieve the set of tuples with balance=1000.
  3. Match the results of steps (1) and (2) to produce a set of tuples that have both branch_name='Bournemouth' and balance=1000.
- **Problem**
  - If there are many tuples in (1) and (2) but few tuples containing both branch_name='Bournemouth' and balance=1000, step 3 will involve searching many records (a sort/merge).

Ref: Silberschatz, sec 11.9
Strategies for Indexing 4

- Given a query:
  
  ```sql
  SELECT * 
  FROM account 
  WHERE branch_name = 'Bournemouth' 
  AND balance = 1000;
  ```

- Strategy:
  1. Use index on \((branch\_name, balance)\) to search for
     \(branch\_name = 'Bournemouth'\) and \(balance = 1000\)

- The query is a partial match query looking for all accounts in the Bournemouth branch with a balance of 1000.
- The fourth strategy for answering this query is to use an index that has been built using the key \((branch\_name, balance)\).
- A B+-Tree index on a composite key is the equivalent of indexing a key constructed from the concatenation of both\(branch\_name\) and \(balance\).
- Given the query above, an index on the composite key \((branch\_name, balance)\) will always identify a matching set of tuples.

Ref: Silberschatz, sec 11.9

Overview

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- Composite Key Index
- Visualising the Index Structure
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- Bitmap Index
A B+-Tree on a composite key indexes the key values as if the composite key
was a single key.

For example, in the B+-Tree above, (Bournemouth, 1000) is less than or equal
to (Bournemouth, 1000) and so it appears in the first leaf node. However,
(Bournemouth, 7500) is greater than (Bournemouth, 1000) and so it appears in
the second leaf node.

The order of the individual attributes in the composite key is important. The
same as sorting a sentence of words alphabetically.

“The ordering of the search-key values is the lexicographic ordering. For
example, for the case of two attribute search keys, (a1,a2) < (b1,b2) if either a1
< b1, or a1 = b1 and a2 < b2.” [Silberschatz et al]

For example, although the second value of (Armagh, 1500) is greater than the
second value of (Bournemouth, 1000), the order of the attributes means that
(Armagh, 1500) is less than (Bournemouth, 1000).

Therefore, the above B+-Tree may be used to search for (branch_name) or
(branch_name, balance) but not (balance). For example, balance=2000
appears in two paths of the B+-Tree.

Ref: Silberschatz, sec 11.9
Visualising a Single-Attribute Index

One Dimension

| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

Splitting the data into blocks can only be done on one dimension

- A B+-Tree is a single-attribute, single-key, index. That is, it can only index one attribute or a concatenation of attributes.
- This may be thought of as a one-dimensional index.
- The data is ordered on a single dimension and the B+-Tree splits the dimension into a sequence of blocks.
- Therefore, the order of the attributes in a composite index is important.

Visualising a Multi-Attribute Index

Two Dimensions

- A multi-attribute index has many dimensions. In the example above there are two dimensions.
- If an index was constructed using two dimensions, then searching for Attribute1 = 100 would only require the Attribute1=100 dimension to be searched.
- The problem is how to divide this two dimensional (or n-dimensional) space into a series of blocks that can be stored on the disc.
- Every point on the grid could be a single block but then only one value would be in each block.
- The two-dimensional space must be partitioned into one or more blocks that contain as many keys as possible.
Partitioning a Multi-Attribute Index

Two Dimensions

- Ideally, the space in a multi-attribute index should be partitioned into the fewest blocks required to store the key values.
- In the above example, there are two blocks. Each block contains two record pointers.
- However, we need a method of deciding how to partition the space and how to track which records are in which blocks.

Overview

- Query Types
  - Exact Match
  - Partial Match
  - Range Match
- Strategies for Querying and Indexing
- Composite Key Index
- Visualising the Index Structure
- Multi-Attribute Index
  - Grid File
- Bitmap Index
A grid file consists of three components:
- A series of blocks on disc that contain the data records (4).
- A multi-dimensional array of pointers (3). A grid of pointers to disc blocks is stored. Each position in the grid corresponds to one combination of attribute values. More than one position in the grid may point to the same disc block. In the above example, positions (4,3), (4,4), (5,3), (5,4), (6,3) and (6,4) all point to block 1.
- An array of key values and pointers for each attribute (1 & 2). The values of each attribute is stored in an array (similar to the directory in an extendible hashing index) together with a pointer. The pointer points to a row or column in the grid. In the above example, the first location in the ‘branch_name’ array (1) points to row 0 in the grid.

The above example shows a grid file with two attributes. However, a grid file may have any number of attributes. Each additional attribute requires an additional array and a new dimension for the grid.

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Searching a grid file involves searching each attribute array, finding the corresponding positions in the grid array and reading the blocks identified by the grid array.

Ref: Silberschatz, sec 11.9.1
Searching a Grid File

Query: ‘Find all records in the Norwich branch.’

1. Search branch_name array for key greater than or equal to ‘Norwich’. Points to row 2 in grid.
2. Search all the blocks pointed to by row 2 in the grid array (X).
3. May require more than one block to be read from disc.

Unlike a one-dimensional index (e.g. the B+-Tree), a grid file can be used to answer queries on any subset of attributes in the index.

In above example, the index is used to search on the branch_name only. The number of blocks that have to be read from disc is reduced to only those blocks pointed to by row 2 in the grid array (i.e. three blocks).

As with the B+-Tree the grid file can be used to perform sequential searches. For example, searching for all balances less than 3000 would search blocks pointed to in columns 0, 1 and 2.

Ref: Silberschatz, sec 11.9.1

Grid File

- Advantages
  - Allows multiple attributes to be indexed.
  - Allows exact match queries.
  - Allows partial match queries.
  - Allows range queries.

- Disadvantages
  - Requires additional space to store arrays.
  - Insertion and deletion can be difficult.
  - When blocks are full, the grid array must be reorganised.

The main advantage of the grid file is that it supports exact match queries, partial match queries and range queries.

This means that a single grid file can replace a number of ordinary indexes.

However, the grid file is more complex than normal indexes and requires more space to store the various arrays and grids.

When a block is full it must be split. Splitting a block requires the pointers in the grid array to be reorganised. Selecting the point to split a block can be difficult as the block may have to be split on more than one attribute.

It is important that the keys in the index have uniform values.
### Overview

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  - Visualising the Index Structure
  - Multi-Attribute Index
    - K-D Tree
  - Bitmap Index

### K-D Tree

A K-D Tree is a multi-attribute index that builds a binary tree. Each split in the tree divides one of the attributes in half.

In the above example, the tree is first split at \(\text{attribute1} = 70\). All keys that are less than 70 are in the left-hand tree and all keys that are greater than or equal to 70 are in the right-hand tree.

The K-D Tree is dynamic. When a block is full it is split by taking the midpoint of one of the attributes. To provide better distribution of the data in the tree, each level on the K-D Tree splits on a different attribute.
Exact Match Search in a K-D Tree

Query: Find all records with Attribute1 = 80 and Attribute2 = 'E'.

- When searching a K-D Tree for an exact match only one path is followed in the tree.

Partial Match Search in a K-D Tree

Query: Find all records with Attribute2 = 'E'.

- When searching a K-D Tree for a partial match more than one path may be followed in the tree.
### K-D Tree

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>- The number of blocks grows as the data is added.</td>
<td>- Special procedures for insert and delete are required.</td>
</tr>
<tr>
<td>- The index adapts to the distribution of values.</td>
<td>- The tree is not balanced.</td>
</tr>
<tr>
<td>- Exact match, partial match and range queries may be performed.</td>
<td></td>
</tr>
</tbody>
</table>

### Overview

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- **Multi-Attribute Index**
  - R-Tree
- **Bitmap Index**
R-Tree - A Spatial Access Method

Key Values are represented as rectangles, for example,

\[ R1 = (x:10, y:60, w:40, h:10) \]

R-Tree - Search

Unlike a B+-Tree, R-Tree nodes can overlap so a search may take many paths down the tree
**R-Tree - Insert**

*Before*

- Ra
- Rb
- R1
- R2
- R3
- R4
- R5
- R6
- R7
- R8
- R9

*After*

- Ra
- Rb
- R1
- R2
- R3
- R4
- R5
- R6
- R7
- R8
- R9

Extend the node that requires the smallest change to include the new key

---

**R-Tree - Node Full & Split**

*Before*

- Ra
- Rb
- R1
- R2
- R3
- R4
- R5
- R6
- R7
- R8
- R9

*After*

- Ra
- Rb
- R1
- R2
- R3
- R4
- R5
- R6
- R7
- R8
- R9
- R10

Extend the node that requires the smallest change to include the new key
**R-Tree**

- **Advantages**
  - Tree structure
  - Similar to B^+-Tree
  - Good search performance
  - Relatively simple to implement
  - Lots of variations
    - R^+-Tree
    - R^*-Tree

- **Disadvantages**
  - Not always appropriate for alphanumeric data
    - Designed for spatial data
      - Map co-ordinates
  - Nodes can overlap
    - Searches can cover more than one path
  - Split algorithms can be complex

---

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  - Multi-Attribute Index
    - Partitioned Hashing

- **Index Types**
  - Bitmap Index
Partitioned Hashing

<table>
<thead>
<tr>
<th>Values</th>
<th>Hash</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Bournemouth, 1000)</td>
<td>10 10</td>
</tr>
<tr>
<td>(Downtown, 1200)</td>
<td>10 10</td>
</tr>
<tr>
<td>(Manchester, 1500)</td>
<td>10 00</td>
</tr>
</tbody>
</table>

Hash function returns binary code with two parts.

A partition hash function takes one or more attributes and returns a sequence of binary bits.

In the example above, the binary code for each set of attributes values has two parts - one for each attribute value. When the bits are combined they can be used to insert the key values in an extendible hashing index.

When querying, if only one attribute value is known, then those entries in the extendible hashing index matching the sequence of known binary bits are searched.

For example, if the query requires branches in Bournemouth, then using the partition hash function produces 10 for Bournemouth and all entries in the directory beginning with 10 are searched. If the query requires balances of 1500, then using the partition hash function produces 00 and all entries in the directory ending in 00 are searched.

The partitioned hash function cannot be used for range searching.

Ref: Silberschatz, sec 11.9.2

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**Table Suitable for Bitmap Index**

<table>
<thead>
<tr>
<th>CUST #</th>
<th>MARITAL</th>
<th>REGION</th>
<th>GENDER</th>
<th>INCOME_ LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>single</td>
<td>east</td>
<td>male</td>
<td>bracket_1</td>
</tr>
<tr>
<td>102</td>
<td>married</td>
<td>central</td>
<td>female</td>
<td>bracket_4</td>
</tr>
<tr>
<td>103</td>
<td>married</td>
<td>west</td>
<td>female</td>
<td>bracket_2</td>
</tr>
<tr>
<td>104</td>
<td>divorced</td>
<td>west</td>
<td>male</td>
<td>bracket_4</td>
</tr>
<tr>
<td>105</td>
<td>single</td>
<td>central</td>
<td>female</td>
<td>bracket_2</td>
</tr>
<tr>
<td>106</td>
<td>married</td>
<td>central</td>
<td>female</td>
<td>bracket_3</td>
</tr>
</tbody>
</table>

*Most attributes have low cardinality*

A B+-Tree would be very large on these attributes and not very efficient.

**Bitmap Index on Region**

<table>
<thead>
<tr>
<th>Cust#</th>
<th>REGION='east'</th>
<th>REGION='central'</th>
<th>REGION='west'</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>102</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>103</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>104</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>105</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>106</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

*A bit is 1 if the tuple has the value represented by the column.*

Very little space is required to store these bits.
### Executing a Query

```sql
SELECT COUNT(*)
FROM CUSTOMER
WHERE MARITAL_STATUS = 'married'
AND REGION IN ('central', 'west');
```

Very efficient bit matching

### Star Schema

```
<table>
<thead>
<tr>
<th>Month</th>
<th>Product</th>
<th>Area</th>
<th>Budget</th>
<th>Range</th>
<th>£</th>
</tr>
</thead>
</table>
```